Analyzing Rational Functions

These notes are intended as a summary of section 2.3 (p. 105 – 112) in your workbook. You should also read the section for more complete explanations and additional examples.

Rational Functions

A rational function has the form

$$y = \frac{f(x)}{g(x)}$$

where $f(x)$ and $g(x)$ are polynomial functions and $g(x) \neq 0$.

The graphs of rational functions have a number of properties that can be determined from their equations. These properties are described below.

Non-Permissible Values of *x*

Since the denominator of a rational function cannot be zero, any value of *x* that would make the denominator zero is not permitted.

If the numerator and denominator of a rational function have a common factor, and the power of the factor in the numerator is equal to or greater than the power of the factor in the denominator, then the graph of the rational function will have a **point of discontinuity** (aka. a **hole**) at the non-permissible value of *x*.

$$y = \frac{(x-3)(x+1)}{(x-3)} \text{ will have a hole at } x = 3$$

If the numerator and denominator of a rational function have a common factor, and the power of the factor in the numerator is less than the power of the factor in the denominator, then the graph of the rational function will have a vertical asymptote at the non-permissible value of *x*.

$$y = \frac{(x+3)(x-3)}{(x-3)^2(x+1)} \text{ will have vertical asymptotes at } x = 3 \text{ and at } x = -1$$

If the numerator and denominator of a rational function do not have a common factor, then the graph of the rational function will have a vertical asymptote at the non-permissible value of *x*.

$$y = \frac{(x+3)}{(x-3)(x+1)} \text{ will have vertical asymptotes at } x = 3 \text{ and at } x = -1$$

Example 1 (sidebar p. 107)
Use the equation of each function to predict whether its graph has vertical asymptotes or holes. Use graphing technology to verify.

a) $y = \dfrac{x^2 - 25}{x + 5}$

b) $y = \dfrac{4x + 2}{x - 2}$

Horizontal Asymptotes

If the numerator and denominator of a rational function have no common factors, then the function will have a horizontal asymptote under the following conditions:

If the degree of the numerator is less than the degree of the denominator, then a rational function will have a horizontal asymptote at $y = 0$.

$$y = \frac{-4}{x+4} \text{ will have a horizontal asymptote at } y = 0$$

If the degrees of the numerator and denominator are equal, then a rational function will have a horizontal asymptote at

$$y = \frac{a}{b}$$

where a is the leading coefficient of the numerator, and b is the leading coefficient of the denominator.

$$y = \frac{2x-3}{x-1} \text{ will have a horizontal asymptote at } y = 2$$

Oblique Asymptotes

If the numerator and denominator of a rational function have no common factors, then the function will have an oblique asymptote under the following conditions:

If the degree of the numerator is 1 more than the degree of the denominator, then a rational function will have an oblique asymptote.

The equation of the oblique asymptote is determined by dividing the numerator by the denominator. The resulting quotient (ignore the remainder) is the equation of the oblique asymptote.

$$y = \frac{x^2}{x+1} \text{ will have an oblique asymptote at } y = x - 1$$

Example 2 (sidebar p. 109)
Use the equation of each function to predict whether its graph has horizontal or oblique asymptotes. Write the equations of these asymptotes. Use graphing technology to verify.

a) $y = \dfrac{x^2 + 6x - 7}{x + 2}$

b) $y = \dfrac{x + 2}{x^2 + 6x - 7}$

Example 3 (sidebar p. 110)
For the graph of each function below

i) Determine any non-permissible values of x, and whether each indicates a hole or a vertical asymptote.
ii) Determine the equations of any horizontal or oblique asymptotes.
iii) Determine the domain.

Use graphing technology to verify the characteristics.

a) $y = \dfrac{x-4}{x^2-4}$

b) $y=\dfrac{x^2-4}{x^2-9}$

Example 4 (sidebar p. 112)
Use graphing technology to solve:

$$\frac{8}{x^2+1}=x$$

Give the solution to the nearest tenth.

Homework: #5 – 10 in the section 2.3 exercises (p. 114 – 121). Answers on p. 122.